IST-2000-29239 PAM
PERFORMANCE ASSESSMENT AND BENCHMARKING OF CONTROLS
2001 – 2004

Weighting Selection for Controller Benchmarking and Tuning

M J Grimble and Pawel Majecki

ICC/**219**/Dec 2004

PAM Document Code: PAM-12-TN-1-V1

Industrial Control Centre
University of Strathclyde
Graham Hills Building
50 George Street
Glasgow, G1 1QE

Telephone: 0141 548 2378/2880
Fax: 0141 548 4203
E-mail: m.grimble@eee.strath.ac.uk
Website: www.icc.strath.ac.uk/

Date: 6 December 2004

Version History

Version	Date	Author(s) and Company	Approved By and Company
1	06/12/2004	Industrial Control Centre, Strathclyde University	Project Steering Committee

Changes from the Previous Authorised Version

Section(s)	Description of Change(s)
All	Original. Not applicable.

Document Control Information

☑ **Controlled Copy No. 1**
☐ **Uncontrolled Copy. For information only.**

Controlled Copy Holders

1. Industrial Systems and Control Limited
2. ABB
3. SIEMENS
4. University of Strathclyde
5. CESI
6. BASF

Summary

The selection of a cost function for benchmarking and performance assessment is always problematic. A new method is described in this report that enables such weighting functions and cost index to be specified knowing the existing control structure. It is assumed that the system already has a rudimentary control algorithm, such as a *PID design* and the requirement is to assess this against a higher order optimal control solution. General guidelines are also discussed.

Contents

1. Weighting selection based on the existing controller structure

A cost function weighting selection method is developed based on a result from *Generalised Minimum Variance* (*GMV*) control laws that may also be extended to Non-linear Generalised Minimum Variance (*NGMV*) results. However, it is also applicable in many cases to *Linear Quadratic Gaussian* (*LQG*) cost weighting selection methods. It may easily be shown that with reasonable choices of dynamic cost function weightings, a *GMV* controller gives similar responses to those of an *LQG* controller. Thus, a weighting selection method that works for *GMV* (or *NGMV*) designs will also apply to *LQG* solutions [1]-[5].

If a system is already controller by a *PID* controller, or some other well defined classical control structure, then a starting choice of *GMV* cost function weighting is to choose the ratio of the error weighting divided by the control weighting equal to the aforementioned controller. There are several assumptions to make this result valid but it is a starting point for design. Moreover, it has realistic frequency response characteristics for weightings inherent in the approach. For example, a *PID* controller clearly has high gain at low frequency and if it includes a filter then it will have low gain at high frequencies. This is exactly the type of response needed for the ratio between the error and control weightings.

The *GMV* procedure is therefore to use the transfer of the existing controller to define the cost function weightings. An indirect benefit of this approach is that it is always difficult to sort out the type of scaling required for defining the cost weightings. Clearly, a system which has different physical parameters will require different cost function weightings, even though the underlying process is the same. By utilising the existing controller structure to define the cost weightings this scaling problem is avoided. Moreover, the type of transient response characteristics obtained for the unmodified optimal control solution will probably be of the same order of those for the classical design. This therefore provides a starting point for weighting selection [7]-[9].

If the system is new and does not have an existing controller then a different procedure must be followed. Such a procedure will require more iterations or a simulation model being available. In this case, the form of the error weighting and the control weighting will probably be defined beforehand but the actual size of the cost function weightings will depend upon the speed of response required from the system. If the system is to be made faster then the magnitude of the control weighting should be reduced. One method of getting into the ball-park of a good solution is to try small control weightings and then a much larger control weighting and interpolate between the two to obtain the type of response required. For example, if the small control weighting gives a one-second response and the large control weighting gives a 50-second response then something in between should give an intermediate value for the dominant time constant. Such a procedure does of course require iteration and on some systems, it will not be possible to try low control weightings that might lead to very harsh actuator movements. Nevertheless, cost-weighting selection can be achieved by such an iterative process.

Out of the two aforementioned methods, the former is the most promising since it provides a very fast way of generating the desired cost weighting functions. Once the existing controller structure is known then the required weightings follow almost immediately. It is true that some adjustment may be necessary after this initial selection, since it is generally the case that the magnitude of the control weighting function needs to be reduced to speed up the system. In this way, the initial design will normally be close to the existing classical controller but the design can be much improved by reducing the value of the control weighting term. Since the initial design will probably give reasonable responses this procedure reduces the danger of any experiments on the plant. In fact, two or three more trials for different weightings will probably be sufficient.

As mentioned above the basic design procedure applies to *GMV* and *NGMV* methods but they also apply to *LQG* solutions by implication and even provide a starting point for weighting selection in H_∞ designs.

2. Relationship to the Smith Predictor

The *GMV* optimal controller can be expressed in a similar form to that of a *Smith Predictor* (Grimble 2001 [8]). It also provides an optimal method of design and optimal stochastic disturbance rejection and tracking properties. However, the use of this structure also limits the applications to open-loop stable systems. That is, although the structure illustrates a useful link between the new solution and the *Smith* time delay compensator, it also has the same disadvantage, that it may only be used on open-loop stable systems. Nevertheless the structure is intuitively reasonable and should be valuable in applications. This *Smith Predictor* will now be explained for open loop stable processes.

It is shown in Grimble 2003 [10, 11] that the GMV controller may be implemented in a Smith Predictor structure. If the system is in the state space model form, the structure is as shown in Fig. 1, which is intuitively reasonable and easy to explain. Note from the control signal u to the feedback signal p that the transfer is null when the model $z^{-k}\mathrm{W}_k$ matches the plant model. It follows that the control action, due to reference signal r changes, is not due to feedback but involves the open-loop stable compensator and the *inner* feedback loop.

This inner-loop has the ratio of the error to control weightings $F_{ck}^{-1}P_c$ acting like an inner-loop controller, with return difference operator: $\left(I - F_{ck}^{-1}P_c\mathrm{W}_k\right)$. Thus, if the plant already has a *PID* controller that stabilises the delay free plant model, the weightings can be chosen equal to the *PID* controller. The choice of the weightings to be equal to a *PID* control law is only a starting point for design, since stability is easier to achieve. However, the control weighting will normally require an additional lead term (or alternatively a high frequency lag term may be added to the error P_c weighting). The high frequency characteristics of the optimal controller will then have a more realistic roll off. This may not be necessary if the *PID* solution already has a low pass filter for noise attenuation.

The implication of these results is that the *GMV* cost weights may be defined in this manner and by implication it will probably be a good starting point for *LQG* weightings.

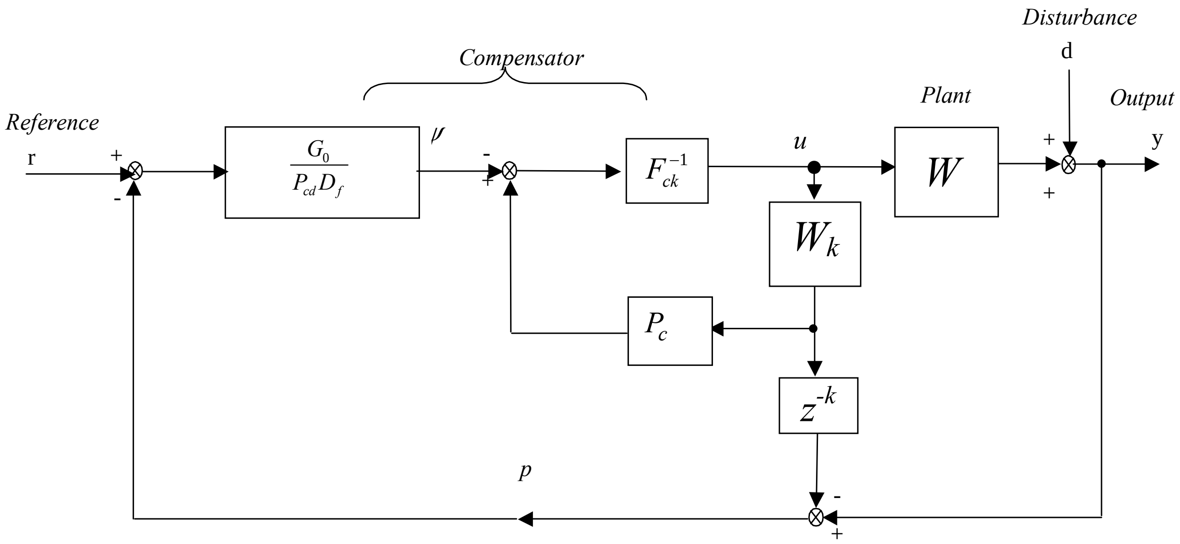

Fig. 1: Nonlinear Smith Predictor Compensator and Internal Model Structure

3. Iterative Cost Function Weighting Selection for Scalar Systems

If the system does not already have a stabilising control solution then the more general guidelines given below must be followed. The selection of cost weighting functions does not involve precise rules but engineering judgements. It is difficult to give rules which ensure a given behaviour is obtained, since in most cases a number of criteria must be satisfied at the same time and trade-offs must be made. The following guidelines will however provide a basis for selecting and changing cost function weightings for $GMV\ /\ H_2\ /\ LQG$ and even H_∞ problems [6].

1. ***Error weighting:*** An integrator on the error weighting function will often result in integral action in the controller. There are a few cases where integral action is not introduced automatically when integral error weightings are used. For example, when two degrees of freedom designs are considered, inferential control is used or when noise models cause a change in the controller response so that pure integral action is not included. The general effect of introducing integral error weighting is, however, to introduce high gain into the controller at low frequencies. This result is also valid for more general disturbance models. If say a system has dominantly sinusoidal disturbances of frequency ω_q, then the weighting can include a lightly damped second order system with natural frequency ω_0. In other words the error weighting

should have a high gain in the frequency range where the disturbances dominate or good tracking accuracy is required.

2. ***Sensitivity weighting:*** When integral weighting is used on the sensitivity function, this has a similar effect to Case 1. Sensitivity costing normally arises in mixed sensitivity problems where measurement noise is not present in the system description and hence integral action in the controller normally occurs (again not necessarily for *2 DOF* or inferential control problems). For more general weightings the sensitivity will normally be reduced in frequency ranges where the magnitude of the weighting is large.

3. ***Lead terms on the control weighting:*** By introducing a high gain at high frequencies on the control weighting term, the controller is normally made to roll off in the frequency range where the gain is high (relative to error weighting terms). The use of a weighting function with high gain at high frequency is more important in H_∞ design than in GMV/H_2 minimisation problems. This weighting provides one mechanism of ensuring the controller will roll off at high frequencies. It ensures the usual wide bandwidth property of H_∞ designs does not lead to unacceptable measurement noise amplification problems. Controller roll-off at high frequencies occurs naturally in *LQG* or H_2 designs due to the use of a measurement noise model. If a measurement noise model is not included, *GMV* and *LQG* designs can give too high a gain at high frequencies.

4. ***Lead terms on the control sensitivity costing:*** The control sensitivity function plays a similar role to the control weighting term referred to in Case 3. In mixed sensitivity problems where a *control sensitivity* term is present, high weighting gain at high frequency is normally advisable for H_∞ designs.

5. ***Complementary sensitivity costing:*** In H_2 or *LQG* problems complementary sensitivity terms are not normally present. In early H_∞ designs these terms were introduced commonly, but the disadvantages have recently been recognised. Complementary sensitivity weighting has an identical effect to combining a weighting function together with the plant transfer function acting on a *control sensitivity* term. Multiplying the control sensitivity function by the plant transfer function does of course give the complementary sensitivity function. Since there are generally disadvantages in using a complementary sensitivity weighting, this term is normally neglected.

6. ***Effects of the weighting functions on the cross-over frequency:*** When large or small error weightings are discussed, this is of course relative to the size of the control weighting terms. In

this context *large* is only in relation to the other weighting functions. Although the weighting functions do have an effect which depends upon the scaling of the system model, it is also true that the point at which the frequency response plots of the error weighting (sensitivity weighting) and the control weighting (control sensitivity weighting) cross often determines the bandwidth point for the system. Indeed a starting point in H_∞ design, for choosing the relative gain sizes, is to choose the cross-over point to coincide with the desired bandwidth.

7. ***Angle between the weightings:*** In general the angle between the frequency responses of the weighting should be limited at the crossover point. Recall that this point is often close to the unity-gain crossover frequency for the system, and the weightings should not therefore introduce rapid unnecessary phase changes unless this is important for stability.

8. ***Lead terms on the error weighting:*** A lead term can be introduced on the error weighting function or sensitivity weighting function in an attempt to improve transient responses. If integral action is used on the error term and a lead term is used on the control weighting, the crossover of the magnitude diagrams will involve a difference in slope of 40 dB per decade. This can result in the system being particularly sensitive in the mid-frequency range. By adding a lead term on the error weighting function, the change in slope can be made 20 dB's per decade and the resulting more gradual phase shifts often lead to a design with better step response characteristics. Similar remarks apply to sensitivity weighting functions where a lead term on the cost weighting may be necessary to reduce the rate of change of gain and phase in the mid- frequency region.

9. ***Robustness weighting function:*** Instead of penalising each of the cost terms independently, it is sometimes more beneficial to multiply each term by the same weighting function: W_σ. This is particularly true when trying to reduce the peak level on sensitivity functions which occur in the mid-frequency range. At low frequencies a high penalty on the sensitivity function will cause a high controller gain which results in a small sensitivity function magnitude. In the frequency range where the loop gain has a magnitude of approximately unity, this rule (that heavy penalties will force down the sensitivity function magnitude) no longer holds. A more effective way of reducing the peak on the sensitivity function, in this case, is to reduce the loop gain so that a frequency response peak of greater than unity does not occur.

   A heavy penalty on the control weighting term or controller sensitivity function can cause a reduction in controller gain and hence an improvement in the sensitivity function in the mid frequency range. The combination of weights which are needed is, however, difficult to determine since in this frequency region the system characteristics are particularly sensitive. Experience has revealed that by using a common weighting function, W_σ, both objectives are met, sometimes providing improvements in both the *sensitivity function* and the *control sensitivity function* simultaneously.

4. Practical selection of LQG and GMV Dynamic Weightings.

A more detailed procedure for the selection of the dynamic weightings is presented in this section, which, together with the general guidelines given above, should assist in the selection (design) process. The GMV and LQG cost functions for the SISO case are collected in Table 1 and the notation therein will be used throughout this section.

	GMV cost function	LQG cost function
Time domain	$E[(P_c e + F_c u)^2]$	$E[(H_q e)^2] + E[(H_r u)^2]$
Frequency domain	$J = \frac{1}{2\pi j} \oint_{\lvert z\rvert=1} \{Q_c \Phi_{ee} + R_c \Phi_{uu} + G_c \Phi_{ue} + G_c^* \Phi_{eu}\} \frac{dz}{z}$	$J = \frac{1}{2\pi j} \oint_{\lvert z\rvert=1} \{Q_c \Phi_{ee} + R_c \Phi_{uu}\} \frac{dz}{z}$
Error weighting	$P_c = \frac{P_{cn}}{P_{cd}}, \quad P_{cd}(0) = 1 \quad and \quad P_{cn}(0) \neq 0$	$Q_c = \frac{Q_n}{A_q^* A_q} = H_q^* H_q = \frac{B_q^* B_q}{A_q^* A_q}$
Control weighting	$F_c = \frac{F_{cn}}{F_{cd}}, \quad F_{cd}(0) = 1 \quad and \quad F_{cn} = F_{ck} z^{-k}$	$R_c = \frac{R_n}{A_r^* A_r} = H_r^* H_r = \frac{B_r^* B_r}{A_r^* A_r}$

Table 1. GMV and LQG cost functions

First we state a few general rules that are useful when no a priori knowledge about the controlled process is available.

RULE 1. The weighting choice should be consistent with the existing controller structure – in practice it means that if the existing controller is of the PID type, the error weighting should include an integrator (see RULE 2). Alternatively, the control weighting could be selected to include a delta (a finite difference) operator – in both cases, the resulting optimal controller contains integral action.

RULE 2. The common requirement is that the error weighting P_c (H_q) should normally include an integral term, which corresponds to the integral action in the controller.

$$\Rightarrow P_c = \frac{P_{cn}}{(1 - z^{-1})} \quad \text{or} \quad \Rightarrow H_q = \frac{B_q}{(1 - z^{-1})}$$

P_{cn} (or B_q) may be constant or they may have the form $(1-\alpha z^{-1})$ where $0<\alpha<1$ is a tuning parameter (the larger α, the sooner integral action will be "turned off")

RULE 3. The control weighting F_c can be chosen as a constant or as a lead term to ensure the controller rolls-off at high frequencies and does not amplify the measurement noise.

$$\Rightarrow F_c = \rho \qquad \text{or} \qquad \Rightarrow F_c = \rho(1-\gamma z^{-1})$$

$$\Rightarrow H_r = \rho \qquad \text{or} \qquad \Rightarrow H_r = \rho(1-\gamma z^{-1})$$

where ρ and γ can be considered tuning parameters. In the case of the GMV design, ρ should normally be negative.

The full utilization of the dynamic weightings is only possible when one has the knowledge of the process model linearized around the working point. This can be identified using one of the common methods, however one can also try to approximate the design given only some limited information about the process – below we will present the procedure for selecting the dynamic weightings, which is based on the two following parameters: the dominant time constant of the process and the process gain. This approach is normally valid for process control where the models can often be approximated as first order lags with time delay.

Note: The procedure applies only to open-loop stable plants, for which the unit step response converges to a finite value.

The approximate method of estimating the process gain and dominant time constant is shown in Fig. 2, which represents the unit step response of an overdamped (or slightly underdamped) system:

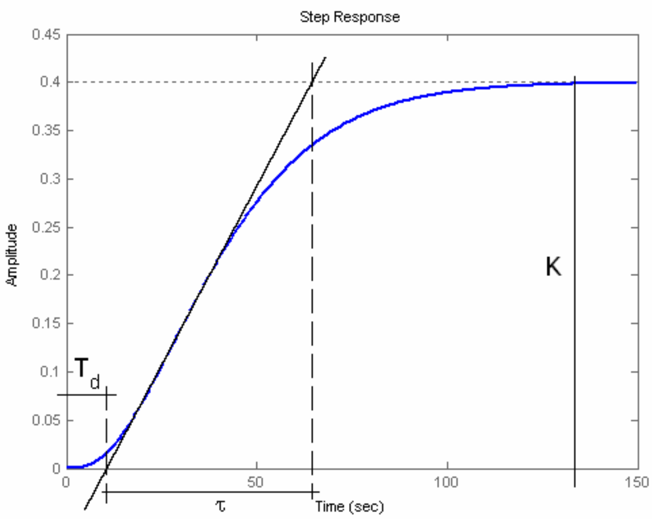

Fig. 2: Time delay plus first-order-lag approximation of an overdamped system

The approximate model is hence assumed to be of the following general form:

$$K(s) = e^{-sT_d} \frac{K}{\tau s + 1}$$

where:

T_d - time delay [s]

τ - dominant time constant [s] (first order equivalent)

K - process gain

The following paragraphs expand on the few rules given above and are concerned with the selection of the LQG weightings in the continuous-time case, given a model of the plant. However, a similar procedure applies also to the selection of GMV weightings (simply replace H_q with P_c and H_r with F_c).

The expression for the LQG cost function above only defines a general form and the crucial step is to define a cost-index which can be used as a benchmark in a particular PID or classical control application. One option is to set Q_c = 1 and R_c = 0, so that a minimum variance benchmark is obtained. The main disadvantage is that unrealistic designs often result that could not be achieved in practice. Some control costing to limit actuator variations is almost always desirable. However, since variance of regulating error can often be related to financial performance, one element in the criterion can involve the variance $E\{e^2(t)\}$.

If the cost index only relates to financial concerns the resulting controller characteristics may be undesirable. To achieve reasonable performance characteristics the following features should normally be included:

(i) The error weighting should include an integrator, so that the controller has integral action to be consistent with a PID restricted structure assumption (in addition to a constant *variance* term).

(ii) In the absence of a measurement noise model, the control weighting should include a lead term to roll off the controller gain at high frequency. This should be consistent with any PID high frequency filters.

The above would suggest a possible parameterization of the dynamic cost weightings, of the form:

$$Q_c = H_q^* H_q \quad \text{and} \quad R_c = H_r^* H_r \tag{1}$$

where

$$H_q = 1 + \omega_q / s \quad \text{and} \quad H_r = \rho(1 + s / \omega_r) \tag{2}$$

Clearly $H_q = (s + \omega_q) / s$ represents integral action, which is cut off at the frequency ω_q. If $\omega_q \to 0$ the minimum variance (constant) term dominates. If ω_c is the desired unity gain crossover frequency for the system, ω_q can be chosen as $\omega_q = \omega_c / 10$ to initiate a design. The resulting H_q has unity gain in the mid to high frequencies and the control weighting must be chosen relative to this value.

The weighting H_r is a lead term and ω_r should be selected to roll-off the controller, where measurement noise dominates. A starting value for a benchmark design is $\omega_r = 10\omega_c$. The value of ρ is chosen to determine the speed of response of the system. The intersection point for the frequency response magnitude plots of $H_q(s)W(s)$ and $H_r(s)$ will be denoted by ω_0 and this frequency is often close to the unity gain crossover frequency for the system. Let ω_g represent the corner frequency for the dominant time constant in the plant model W. Then $\omega_0 \cong \omega_c$ can be chosen to be $\omega_0 = 3\omega_g$ for a process plant and $\omega_0 = 10\omega_g$ for a machine control system.

To determine ρ, the point at which the plots of $H_q(s)W(s)$ and $H_r(s)$ intersect is required. That is,

$$|H_q(j\omega_0)|.|W(j\omega_0)| = |H_r(j\omega_0)|$$

At a particular frequency ω_0 write:

$$H_q(j\omega_0)W(j\omega_0) = H_w^r + jH_w^i$$

and

$$H_r(j\omega_0) = \rho(H_r^r + jH_r^i)$$

Then ρ can be found as the point where

$$\rho^2((H_r^r)^2 + (H_r^i)^2) = (H_w^r)^2 + (H_w^i)^2$$

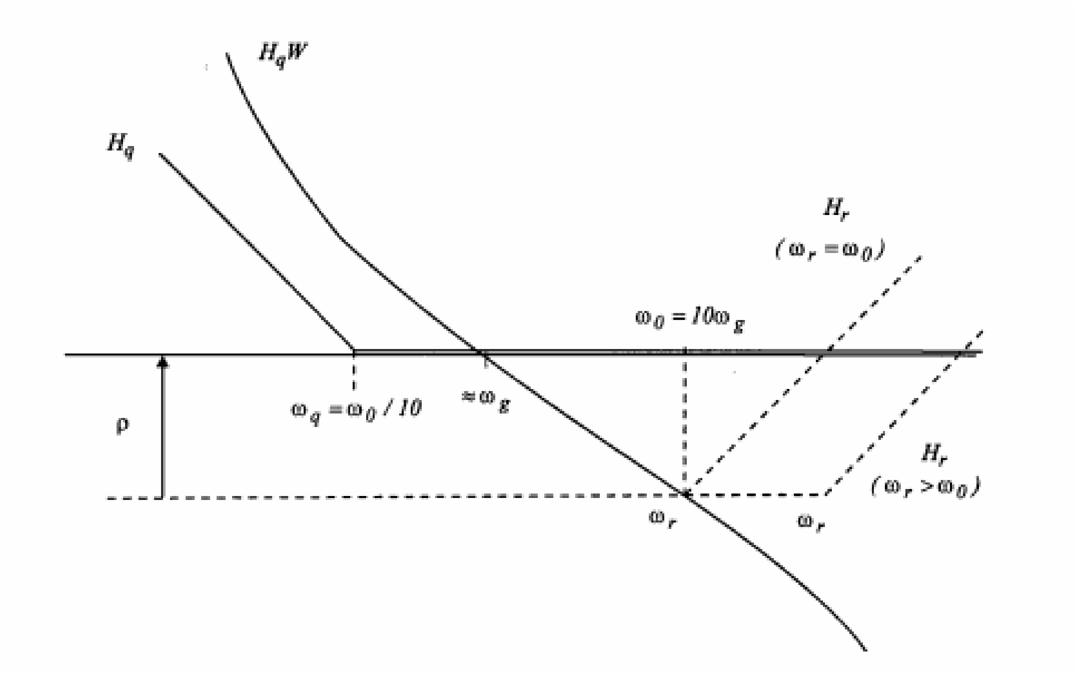

Fig. 3 : Error and Control Weighting Frequency Responses
(Two control weighting choices all with common intersection point at the frequency ω_0)

To summarise the design choices:

(i) Typically $3\omega_g < \omega_0 < 10\omega_g$ and $\omega_c \cong \omega_0$

(ii) $\omega_q = \omega_0 / 10$ and $\omega_r = 10\omega_0$

(iii) $\rho = \sqrt{\dfrac{(H_w^r)^2 + (H_w^i)^2}{(H_r^r)^2 + (H_r^i)^2}}$ evaluated at ω_0 .

As previously noted, another possible and fundamental method of choosing ρ is to specify the desired relationship between input and output powers at the optimum:

$$\eta = E\{e^2(t)\} / E\{u^2(t)\}|_{opt}$$

This completes the specification of the benchmark performance cost against which to judge a PID design.

Remarks:

Only three parameters are needed to fully determine the simple weightings given in (2):

1. Cut-off frequency ω_q [rad/s]
2. Cut-off frequency ω_r [rad/s]
3. Control weighting gain ρ

The first two can be determined given only the dominant process time constant. To find parameter ρ, however, the knowledge of the plant model W is required. If no such accurate model is available, then there are two possible options to consider:

- plant model W is approximated with first order dynamics using the dominant time constant and the gain.
- ρ is considered a tuning parameter determining the relative importance of the variance of the error signal and control activity – hence, the changing of this parameter will modify the speed of response of the system.

The weightings defined in (2) correspond to the continuous-time domain. In order to obtain their discrete-time equivalents, it is necessary to use one of the discretisation methods – if the sampling period is small enough, such approximation will be valid. The common discretisation method is Tustin's rule, which maps s-domain into z-domain according to the formula

$$s = \frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}, \qquad (T_s \text{ - sampling time})$$

which yields the discrete LQG weightings (just replace H_q with P_c and H_r with $-F_{ck}$ to obtain GMV weightings):

$$H_q = 1 + \frac{\omega_q T_s (1+z^{-1})}{2(1-z^{-1})} \qquad \text{and} \qquad H_r = \rho(1 + \frac{2(1-z^{-1})}{\omega_r T_s (1+z^{-1})})$$

To further illustrate the procedure, the algorithm will be presented for the special case of a first-order process with time delay.

Algorithm

Input parameters: T_d, K, τ, β, T_s

β is a number from 3 to 10 – higher numbers indicate faster desired response. For example, β = 3 may correspond to a process plant, and β = 10 to a machine control system. T_s is the sampling period in seconds. Time delay T_d is only necessary to determine the discrete delay k in samples and is not used explicitly in the algorithm.

Outputs: ω_q, ω_r, ρ

(1°) $\omega_g = 2\pi / \tau$ - plant corner frequency [rad/s]

(2°) $\omega_c = \beta\omega_g$

(3°) $\omega_q = \omega_c / 10$ and $\omega_r = 10\omega_c$

(4°) $$\rho = \frac{\omega_r}{K\omega_c}\sqrt{\frac{(\omega_c^2 + \omega_q^2)(\omega_c^2\tau^2 + 1)}{(\omega_c^2 + \omega_r^2)}}$$

5. References

1. Åström, K J, 1979, Introduction to stochastic control theory, Academic Press, London.
2. Hastings-James, R, 1970, A linear stochastic controller for regulation of systems with pure time delay, University of Cambridge, Department of Engineering, Research Report No CN/70/3.
3. Clarke, D W, and Hastings-James, R, 1971, Design of digital controllers for randomly disturbed systems, Proc. IEE, Vol. 118, No. 10, 1502-1506.
4. Grimble, M J, 1981, A control weighted minimum-variance controller for non-minimum phase systems, International J Control, Vol 33, No 4, pp 751-762.
5. Grimble, M J, 1988, Generalized minimum-variance control law revisited, Optimal Control Applications and Methods, Vol. 9, pp 63-77.
6. Grimble, M J, 1993, H_∞ multivariable control law synthesis, IEE Proceedings, Control Theory and Applications, Vol. 140, No 5, pp 353-363.
7. Grimble, M J, 1994, Robust industrial control, Prentice Hall, Hemel Hempstead.
8. Grimble, M J, 2001, Industrial control systems design, John Wiley, Chichester.
9. Grimble, M J, and Johnson, M A, 1988, Optimal multivariable control and estimation theory : theory and applications, Vols. I and II, John Wiley, London.
10. Grimble, M.J., "Generalized Minimum Variance Control of Non-linear Multivariable System", Industrial Control Centre Report, 210, November 2003.
11. Grimble, M.J., "Nonlinear Generalized Minimum Variance Feedback, Feedforward and Tracking Control ", Industrial Control Centre Report, 211, October 2003.